Abstract

SULI 2016, PPPL
Matthew Barry

The optimization of superconducting magnet performance and development of high-field superconducting magnets will greatly impact the next generation of fusion devices. A successful magnet development, however, relies deeply on the understanding of superconducting materials. Among the numerous factors that impact a superconductor's performance, mechanical stress is the most important because of the extreme operation temperature and large electromagnetic forces. Typical superconducting wires consist of a stabilizer, a barrier, a matrix and superconducting filaments. In previous work, J. Chen developed a model to calculate the stress/strain in individual Nb_3Sn filaments embedded in a matrix. In this study, mechanical theory is used to further develop this model by modeling the barrier and stabilizer. Both thermal loads and mechanical loads are included in the analysis to simulate operation conditions. Because the combination of these models will simulate the typical architecture of major superconducting materials, such as Nb_3Sn, MgB_2, Bi-2212 etc., it will provide a good overall picture for us to understand the behavior of these superconductors in terms of thermal and mechanical loads.

Analysis of Mechanical Stresses/Strains in Superconducting Wire

Matthew Barry[1], Jingping Chen[2], Yuhu Zhai[2]
[1]Auburn University, Auburn, AL, [2]Princeton Plasma Physics Lab, Princeton, NJ

This work was made possible by funding from the Department of Energy for the Summer Undergraduate Laboratory Internship (SULI) program. This work is supported by the US DOE Contract No. DE-AC02-09CH11466.

Abstract

The optimization of superconducting magnet performance and development of high-field superconducting magnets will greatly impact the next generation of fusion devices. A successful magnet development, however, relies deeply on the understanding of superconducting materials. Among the numerous factors that impact a superconductor's performance, mechanical stress is the most important because of the extreme operation temperature and large electromagnetic forces. Typical superconducting wires consist of a stabilizer, a barrier, a matrix and superconducting filaments. In previous work, J. Chen developed a model to calculate the stress/strain in individual Nb_3Sn filaments embedded in a matrix [1]. In this study, mechanical theory is used to further develop this model by modeling the barrier and stabilizer. Both thermal loads and mechanical loads are included in the analysis to simulate operation conditions. Because the combination of these models will simulate the typical architecture of major superconducting materials, such as Nb_3Sn, MgB_2, Bi-2212 etc., it will provide a good overall picture for us to understand the behavior of these superconductors in terms of thermal and mechanical loads.

Background

- The superconducting properties of Nb_3Sn are highly strain sensitive
- The strain in individual Nb_3Sn filaments, rather than the overall strain in the wire, influences the superconducting properties
- Previous work in [1] developed a theory that relates the strain in each filament directly to the external load applied to the wire (Fig. 1 and Fig. 2)
- This theory connects critical current performance directly to filament strain (Fig. 3)
- The model in [1], however, only includes the CuSn matrix and Nb_3Sn filaments, neglecting the Nb barrier and Cu stabilizer

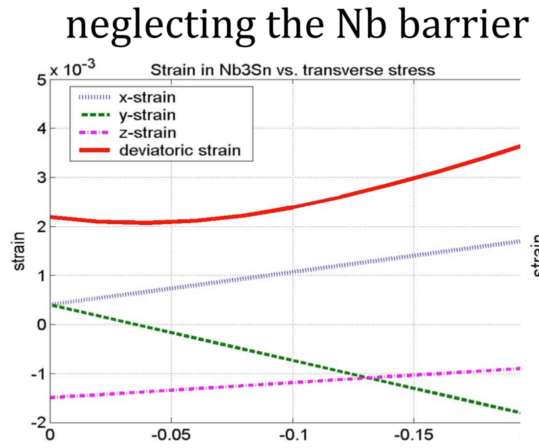


Fig. 1 Strain of Nb_3Sn filament vs. applied transverse compressive strain

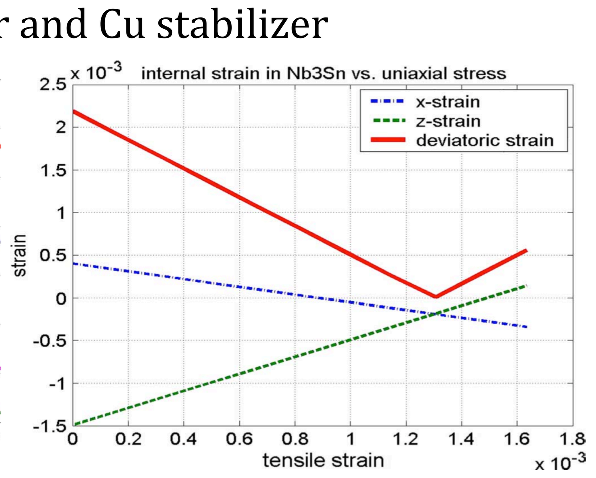


Fig. 2 Strain of Nb_3Sn filament vs. applied tensile strain

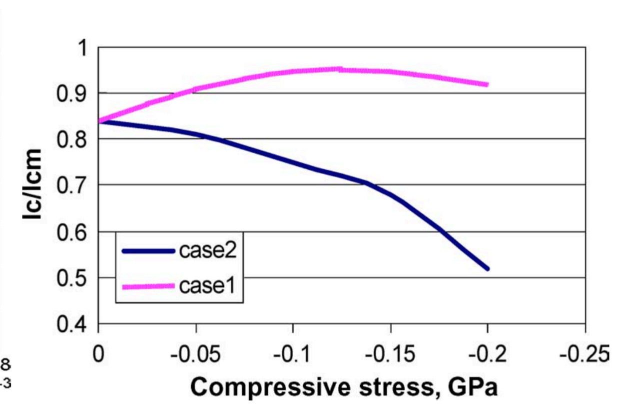


Fig. 3 I_c/I_{cm} vs. compressive stress for two cases of applied load. In each case the load is applied differently to show the dramatic effect it has on performance

Model

Nb_3Sn wire geometry can be modeled as a composite of three nested cylinders where the matrix is a combination of the CuSn shell and network and Nb_3Sn filaments.

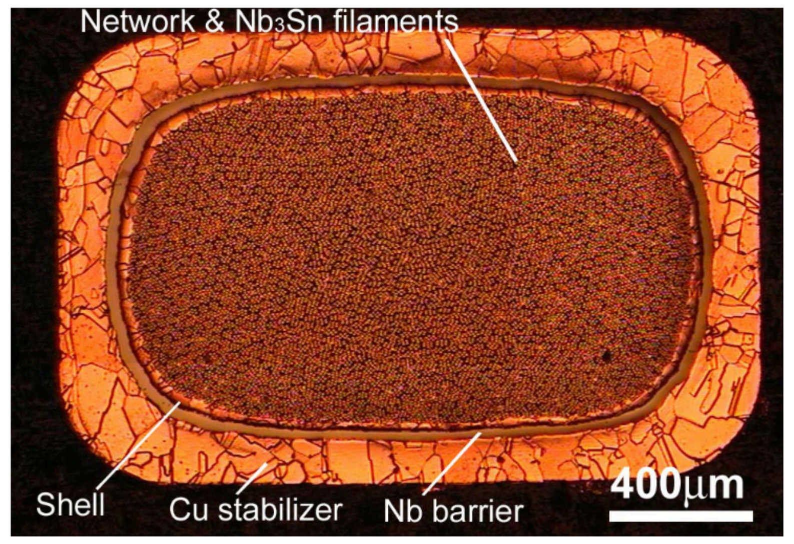


Fig. 4 Cross section of Nb_3Sn superconducting wire. The shell area is CuSn between the bundles of filaments and Nb diffusion barrier, and the network area is the CuSn between bundles of filaments.

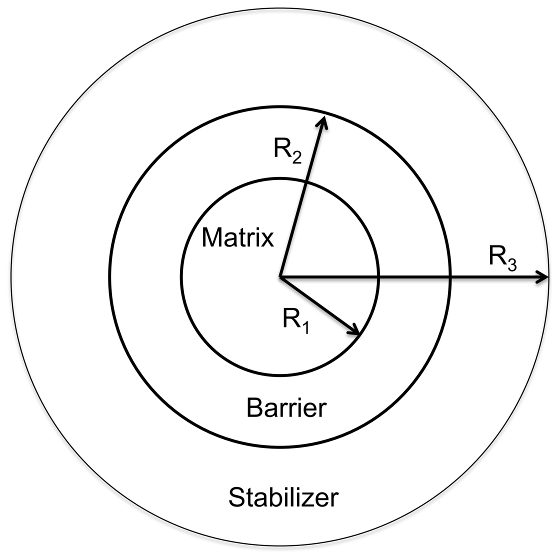


Fig. 5 Cross section of Nb_3Sn superconducting wire represented as a composite of three nested cylinders. The matrix is a combination of the CuSn shell and network and Nb_3Sn filaments.

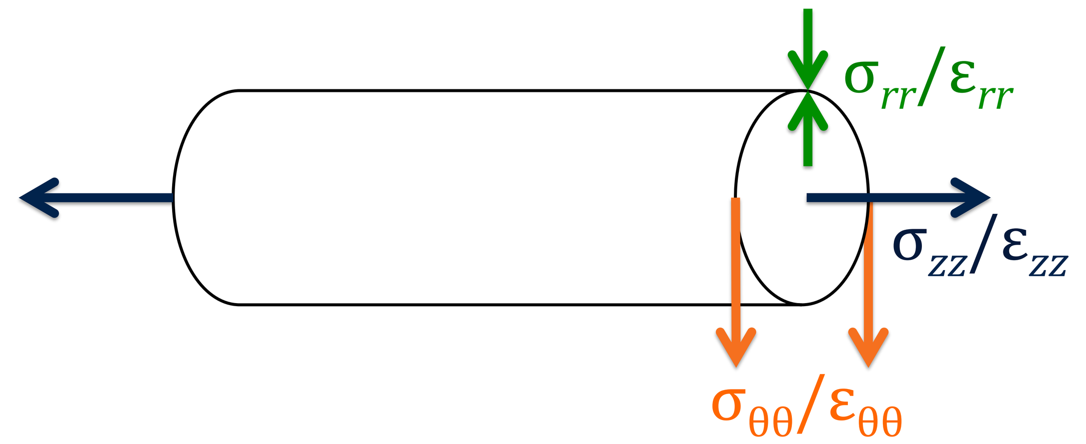
Methodology

Previous work in [2] developed a model to analyze the stresses/strains in two and three cylinder cases due to a temperature change. We furthered this model by adding an applied force term in the longitudinal direction representative of the electromagnetic forces during operation. Both our model and the model presented in [2] are valid for transversal isotropic materials. A MATLAB model was developed rather than using finite element software so that initial conditions could be easily changed and output could be used for further calculations.

- The stress/strain relationship of each element of the composite wire (Matrix, Barrier, and Stabilizer) can be described in the Duhamel-Neumann form:

$$\begin{pmatrix} \varepsilon_{rr} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{zz} \end{pmatrix} = \begin{pmatrix} \frac{1}{E_r} & -\frac{v_{\theta r}}{E_r} & -\frac{v_{zr}}{E_z} \\ -\frac{v_{\theta r}}{E_r} & \frac{1}{E_r} & -\frac{v_{zr}}{E_z} \\ -\frac{v_{zr}}{E_z} & -\frac{v_{zr}}{E_z} & \frac{1}{E_z} \end{pmatrix} \begin{pmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \\ \sigma_{zz} \end{pmatrix} + \begin{pmatrix} \alpha_r \\ \alpha_r \\ \alpha_z \end{pmatrix} \Delta T$$

- Where ε is the strain, σ is the stress, ν is Poisson's ratio, E is the modulus of elasticity, α is the coefficient of thermal expansion, T is temperature, and the *rr* (radial), θθ (hoop), and *zz* (longitudinal) directions are as shown:



- The stress in each element at a distance r from the symmetry axis is represented by:

$$\sigma_{rr} = -P_i \frac{(b/r)^2 - 1}{(b/a)^2 - 1} - P_o \frac{1-(a/r)^2}{1-(a/b)^2} \qquad \sigma_{\theta\theta} = P_i \frac{(b/r)^2 + 1}{(b/a)^2 - 1} - P_o \frac{1+(a/r)^2}{1-(a/b)^2} \qquad \sigma_{zz} = \xi$$

- Where P_i is internal pressure, P_o is external pressure, a and b are the internal and external radii of the element, and ξ is a constant

- The following boundary conditions and continuity relations are applied (three cylinder case is shown):

$$\sigma_{rr}^m = \sigma_{rr}^b \text{ at } r = R_1 \qquad \varepsilon_{zz}^m = \varepsilon_{zz}^b \text{ at } r = R_1$$
$$\sigma_{rr}^b = \sigma_{rr}^s \text{ at } r = R_2 \qquad \varepsilon_{\theta\theta}^m = \varepsilon_{\theta\theta}^b \text{ at } r = R_1$$
$$\sigma_{rr}^s = 0 \text{ at } r = R_3 \qquad \varepsilon_{zz}^s = \varepsilon_{zz}^b \text{ at } r = R_2$$
$$\varepsilon_{\theta\theta}^s = \varepsilon_{\theta\theta}^b \text{ at } r = R_2$$

- A force balance in the zz direction, including the applied stress, is used to solve the system (three cylinder case is shown):

$$0 = (\sigma_{zz}^m)(A^m) + (\sigma_{zz}^b)(A^b) + (\sigma_{zz}^s)(A^s) + (\sigma_{zz}^{app})(A^{total})$$

- The solution to this system yields the longitudinal, hoop, and radial stresses/strains as a function of the radial position on the composite for any temperature change and applied force in the zz direction

Radial and Hoop Stress in CuNb Composite

- Using the two cylinder model, a CuNb composite can be modeled where Nb is the inner cylinder and Cu is the outer cylinder
- The model is 1mm in diameter, 70% Cu, and 30% Nb by volume
- The radial position is normalized where 0 is the Cu-Nb boundary and 1 is the outermost part of the Cu cylinder.
- The radial and hoop stress in the Cu cylinder can be plotted as a function of radial position for different operation conditions (Fig. 6 and Fig. 7)

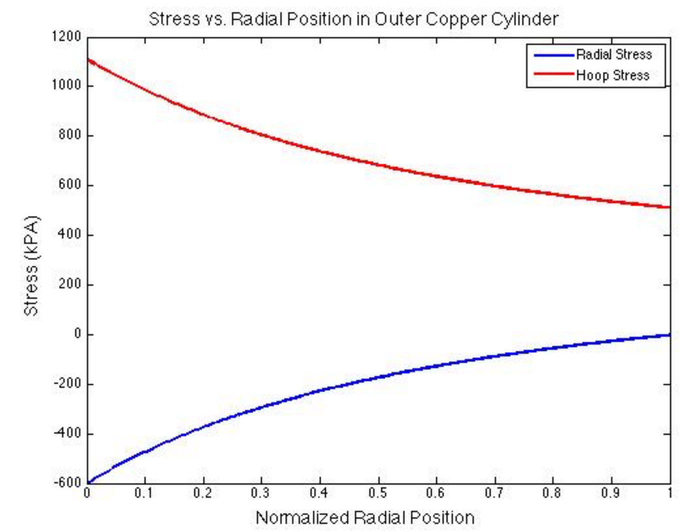


Fig. 6 Radial and hoop stress in the outer Cu cylinder due to a one degree temperature change.

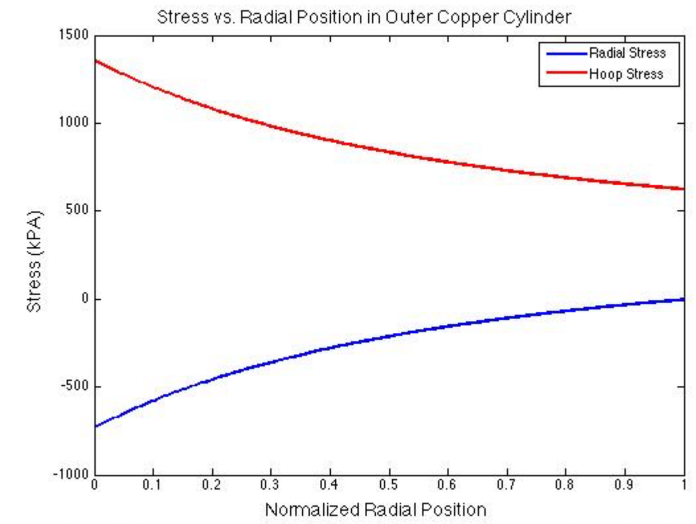


Fig. 7 Radial and hoop stress in the outer Cu cylinder due to a one degree temperature change and a 10N applied force in the longitudinal direction

- The radial and hoop stress in the inner Nb cylinder is constant at all radial positions for a given operation condition

Thermal Expansion of CuNb Composite

- Individually, Cu and Nb have positive thermal expansion coefficients at cryogenic temperatures (normal behavior)
- It has been observed in experiment that as a composite, CuNb has a negative thermal expansion coefficient below 20K (abnormal behavior)
- Our model correctly simulates this irregular thermal expansion behavior as shown in Fig. 8

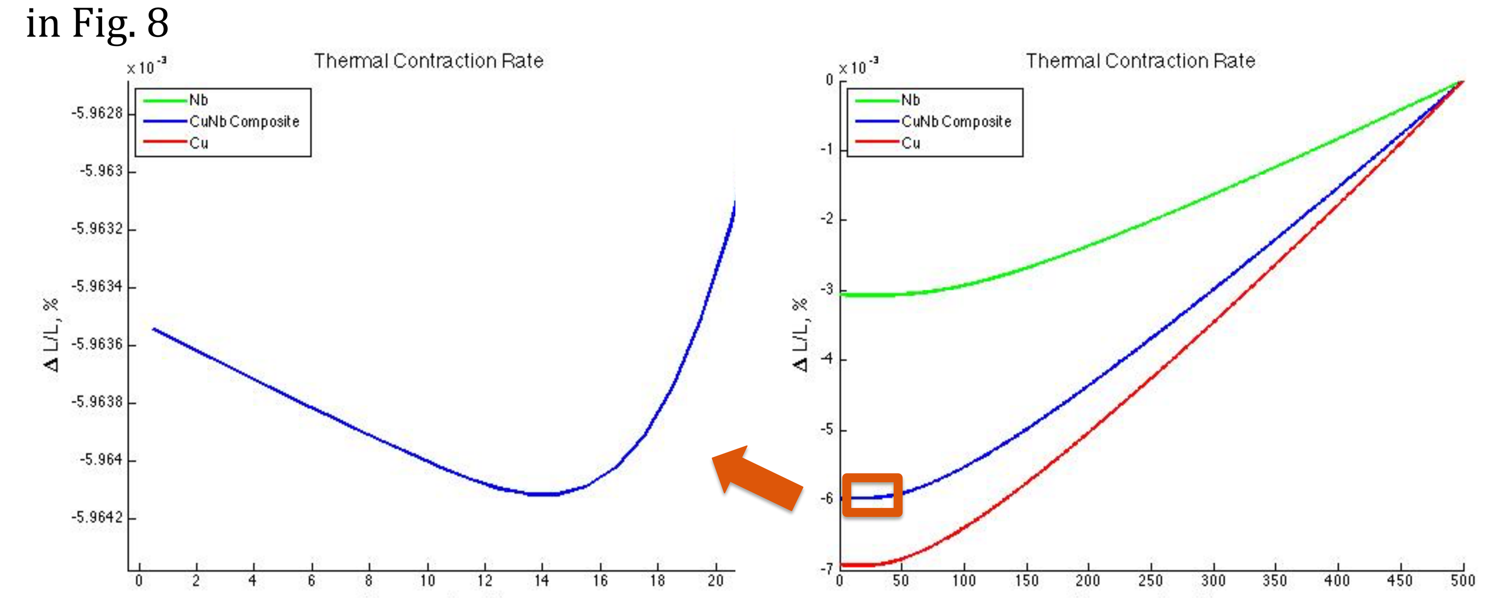


Fig. 8 Thermal contraction rate of CuNb composite (70% Cu and 30% Nb by volume)

Future Work

- The next step is to combine this model with the model in [1] to give a complete representation of typical superconducting wire geometry
- With this combined model, stress/strain distributions in typical superconducting wires can be modeled under various operation conditions.

References

[1] J. Chen, "3D stress-strain model of the Nb_3Sn wire," *IEEE Trans. Appl. Supercond.*, vol. 21, no. 3, pp. 2509-2512, 2011.
[2] D. H. Wagner, "Thermal residual stress in composites with anisotropic interphases," *Phys. Rev. B*, vol. 53, no. 9, pp. 5055-5058, 1996.
[3] J. Chen, "Development and characterization of high strength Nb_3Sn superconductor," Ph.D Thesis, Florida State University, Tallahassee, Florida.